# OCR Maths FP1

# Topic Questions from Papers

# Matrices

**1** The matrices **A** and **I** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

**(i)** Find $\mathbf{A}^2$ and verify that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$. [4]

**(ii)** Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$. [2]

*(Q2, June 2005)*

**2** The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

**(i)** Given that **B** is singular, show that $a = -\frac{2}{3}$. [3]

**(ii)** Given instead that **B** is non-singular, find the inverse matrix $\mathbf{B}^{-1}$. [4]

**(iii)** Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned}$$

[3]

*(Q7, June 2005)*

**3** **(i)** Write down the matrix **C** which represents a stretch, scale factor 2, in the $x$-direction. [2]

**(ii)** The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **D**. [2]

**(iii)** The matrix **M** represents the combined effect of the transformation represented by **C** followed by the transformation represented by **D**. Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}.$$

[2]

*(Q9, June 2005)*

**4** The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

**(i)** Find the value of the determinant of **M**. [3]

**(ii)** State, giving a brief reason, whether **M** is singular or non-singular. [1]

*(Q3, Jan 2006)*

**5** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

**(i)** Find $\mathbf{C}^{-1}$. [2]

**(ii)** Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find $\mathbf{B}^{-1}$. [5]

*(Q6, Jan 2006)*

**6** The matrix $\mathbf{T}$ is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

**(i)** Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{T}$. [3]

**(ii)** The transformation represented by matrix $\mathbf{T}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them. [6]

*(Q8, Jan 2006)*

**7** The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

**(i)** Find $\mathbf{A} + 3\mathbf{B}$. [2]

**(ii)** Show that $\mathbf{A} - \mathbf{B} = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and $k$ is a constant whose value should be stated. [2]

*(Q1, June 2006)*

**8** The transformation S is a shear parallel to the $x$-axis in which the image of the point (1, 1) is the point (0, 1).

**(i)** Draw a diagram showing the image of the unit square under S. [2]

**(ii)** Write down the matrix that represents S. [2]

*(Q2, June 2006)*

**9** The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

**(i)** Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

*(Q7, June 2006)*

**10** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of $\mathbf{M}$. [3]

**(ii)** Hence find the values of $a$ for which $\mathbf{M}$ is singular. [3]

**(iii)** State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay \phantom{{}+ 2z} &= 1, \\ x + 2y + z &= 3, \end{aligned}$$

have any solutions when

**(a)** $a = 3$,

**(b)** $a = 2$.

[4]

*(Q8, June 2006)*

**11** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.

**(i)** Given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$, write down the value of $a$. [1]

**(ii)** Given instead that $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$, find the value of $a$. [2]

*(Q1, Jan 2007)*

**12** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$.

**(i)** Draw a diagram showing the unit square and its image under the transformation represented by **C**. [2]

The transformation represented by **C** is equivalent to a rotation, R, followed by another transformation, S.

**(ii)** Describe fully the rotation R and write down the matrix that represents R. [3]

**(iii)** Describe fully the transformation S and write down the matrix that represents S. [4]

*(Q9, Jan 2007)*

**13** The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$, where $a \neq 2$.

**(i)** Find $\mathbf{D}^{-1}$. [7]

**(ii)** Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y \phantom{{}+2z} &= 3, \\ 3x + y + 2z &= 4, \\ -y + z &= 1. \end{aligned}$$

[4]

*(Q10, Jan 2007)*

**14** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}$.

**(i)** Find $\mathbf{A}^{-1}$. [2]

The matrix $\mathbf{B}^{-1}$ is given by $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$.

**(ii)** Find $(\mathbf{AB})^{-1}$. [4]

*(Q4, June 2007)*

**15** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 0 \\ 0 & a & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of $\mathbf{M}$. [3]

**(ii)** In the case when $a = 2$, state whether $\mathbf{M}$ is singular or non-singular, justifying your answer. [2]

**(iii)** In the case when $a = 4$, determine whether the simultaneous equations

$$\begin{aligned} ax + 4y \qquad &= 6, \\ ay + 4z &= 8, \\ 2x + 3y + \ z &= 1, \end{aligned}$$

have any solutions. [3]

*(Q7, June 2007)*

**16** **(i)** Write down the matrix, $\mathbf{A}$, that represents an enlargement, centre $(0, 0)$, with scale factor $\sqrt{2}$. [1]

**(ii)** The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by $\mathbf{B}$. [3]

**(iii)** Given that $\mathbf{C} = \mathbf{AB}$, show that $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. [1]

**(iv)** Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{C}$. [2]

**(v)** Write down the determinant of $\mathbf{C}$ and explain briefly how this value relates to the transformation represented by $\mathbf{C}$. [2]

*(Q9, June 2007)*

**17** The transformation S is a shear with the $y$-axis invariant (i.e. a shear parallel to the $y$-axis). It is given that the image of the point $(1, 1)$ is the point $(1, 0)$.

**(i)** Draw a diagram showing the image of the unit square under the transformation S. [2]

**(ii)** Write down the matrix that represents S. [2]

*(Q1, Jan 2008)*

**18** The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 4 \quad -1)$. Find

**(i)** $\mathbf{A} - 4\mathbf{B}$, [2]

**(ii)** $\mathbf{BC}$, [4]

**(iii)** $\mathbf{CA}$. [2]

*(Q5, Jan 2008)*

**19** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

**(i)** Given that **A** is singular, find $a$. [2]

**(ii)** Given instead that **A** is non-singular, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned}$$ [5]

*(Q7, Jan 2008)*

**20** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and **I** is the $2 \times 2$ identity matrix. Find

**(i)** $\mathbf{A} - 3\mathbf{I}$, [2]

**(ii)** $\mathbf{A}^{-1}$. [2]

*(Q1, June 2008)*

**21** Describe fully the geometrical transformation represented by each of the following matrices:

**(i)** $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$, [1]

**(ii)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, [2]

**(iii)** $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, [2]

**(iv)** $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$. [2]

*(Q7, June 2008)*

**22** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix **B** is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

**(i)** Show that **AB** is non-singular. [2]

**(ii)** Find $(\mathbf{AB})^{-1}$. [4]

**(iii)** Find $\mathbf{B}^{-1}$. [5]

*(Q10, June 2008)*

**23** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ a & 5 \end{pmatrix}$. Find

**(i)** $\mathbf{A}^{-1}$, **[2]**

**(ii)** $2\mathbf{A} - \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$. **[2]**

*(Q2, Jan 2009)*

**24** Given that **A** and **B** are $2 \times 2$ non-singular matrices and **I** is the $2 \times 2$ identity matrix, simplify

$$\mathbf{B}(\mathbf{AB})^{-1}\mathbf{A} - \mathbf{I}. \quad [4]$$

*(Q4, Jan 2009)*

**25** By using the determinant of an appropriate matrix, or otherwise, find the value of $k$ for which the simultaneous equations

$$\begin{aligned} 2x - y + z &= 7, \\ 3y + z &= 4, \\ x + ky + kz &= 5, \end{aligned}$$

do not have a unique solution for $x$, $y$ and $z$. **[5]**

*(Q5, Jan 2009)*

**26** **(i)** The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of transformation P. **[2]**

**(ii)** The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation Q. **[2]**

**(iii)** The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R. **[2]**

**(iv)** Give a geometrical description of the **single** transformation that is represented by your answer to part **(iii)**. **[3]**

*(Q6, Jan 2009)*

**27** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the $2 \times 2$ identity matrix. Find the values of the constants $a$ and $b$ for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. **[4]**

*(Q2, June 2009)*

**28** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

**(i)** Draw a diagram showing the image of the unit square under the transformation represented by **C**. **[3]**

The transformation represented by **C** is equivalent to a transformation S followed by another transformation T.

**(ii)** Given that S is a shear with the $y$-axis invariant in which the image of the point (1, 1) is (1, 2), write down the matrix that represents S. **[2]**

**(iii)** Find the matrix that represents transformation T and describe fully the transformation T. **[6]**

*(Q8, June 2009)*

**29** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of **A**. **[3]**

**(ii)** Hence find the values of $a$ for which **A** is singular. **[3]**

**(iii)** State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + y + z &= 2a, \\ x + ay + z &= -1, \\ x + y + 2z &= -1, \end{aligned}$$

have any solutions when

**(a)** $a = 0$,

**(b)** $a = 1$.

**[4]**

*(Q9, June 2009)*

**30** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and **I** is the $2 \times 2$ identity matrix.

**(i)** Find $\mathbf{A} - 4\mathbf{I}$. **[2]**

**(ii)** Given that **A** is singular, find the value of $a$. **[3]**

*(Q1, Jan 2010)*

**31** **(i)** The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T. **[2]**

**(ii)** The transformation T is equivalent to a reflection in the line $y = -x$ followed by another transformation S. Give a geometrical description of S and find the matrix that represents S. **[4]**

*(Q5, Jan 2010)*

**32** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

**(i)** Find $\mathbf{A}^{-1}$. **[7]**

**(ii)** Hence, or otherwise, solve the equations

$$\begin{aligned} 2x - y + z &= 1, \\ 3y + z &= 2, \\ x + y + az &= 2. \end{aligned}$$

**[4]**

*(Q9, Jan 2010)*

**33** The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

**(i)** **AB**, **[2]**

**(ii)** **BA** – 4**C**. **[4]**

*(Q2, June 2010)*

**34** **(a)** Write down the matrix that represents a reflection in the line $y = x$. **[2]**

**(b)** Describe fully the geometrical transformation represented by each of the following matrices:

**(i)** $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, **[2]**

**(ii)** $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. **[2]**

*(Q5, June 2010)*

**35** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of **A**. **[3]**

**(ii)** Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of $a$, determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

**(a)** $a = 0$

**(b)** $a = 1$

**(c)** $a = 2$

**[6]**

*(Q9, June 2010)*

**36** The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find

**(i)** $2\mathbf{A} + \mathbf{B}$, **[2]**

**(ii)** **AC**, **[2]**

**(iii)** **CB**. **[3]**

*(Q1, Jan 2011)*

**37** Given that **A** and **B** are non-singular square matrices, simplify

$$\mathbf{AB}(\mathbf{A}^{-1}\mathbf{B})^{-1}.$$ **[3]**

*(Q5, Jan 2011)*

**38** **(i)** Write down the matrix, **A**, that represents a shear with $x$-axis invariant in which the image of the point (1, 1) is (4, 1). **[2]**

**(ii)** The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. **[2]**

**(iii)** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.

**(a)** Draw a diagram showing the unit square and its image under the transformation represented by **C**. **[3]**

**(b)** Write down the determinant of **C** and explain briefly how this value relates to the transformation represented by **C**. **[2]**

*(Q7, Jan 2011)*

**39** The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of **M**. **[3]**

**(ii)** Hence find the values of $a$ for which $\mathbf{M}^{-1}$ does not exist. **[3]**

**(iii)** Determine whether the simultaneous equations

$$6x - 6y + z = 3k,$$
$$3x + 6y + z = 0,$$
$$4x + 2y + z = k,$$

where $k$ is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. **[3]**

*(Q9, Jan 2011)*

**40** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$. **I** denotes the $2 \times 2$ identity matrix. Find

**(i)** $\mathbf{A} + 3\mathbf{B} - 4\mathbf{I}$, **[3]**

**(ii)** $\mathbf{AB}$. **[2]**

*(Q1, June 2011)*

**41** By using the determinant of an appropriate matrix, find the values of $k$ for which the simultaneous equations

$$kx + 8y = 1,$$
$$2x + ky = 3,$$

do not have a unique solution. **[3]**

*(Q3, June 2011)*

**42** The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$, where $a \neq 1$. Find $\mathbf{C}^{-1}$. **[7]**

*(Q6, June 2011)*

**43** The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$.

**(i)** The diagram in the printed answer book shows the unit square $OABC$. The image of the unit square under the transformation represented by $\mathbf{X}$ is $OA'B'C'$. Draw and label $OA'B'C'$. **[3]**

**(ii)** The transformation represented by $\mathbf{X}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them. **[4]**

*(Q8, June 2011)*

**44** The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and $\mathbf{I}$ is the $2 \times 2$ identity matrix.

Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants $p$ and $q$. **[5]**

*(Q2, Jan 2012)*

**45** **(a)** Find the matrix that represents a reflection in the line $y = -x$. **[2]**

**(b)** The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

**(i)** Describe fully the geometrical transformation represented by $\mathbf{C}$. **[2]**

**(ii)** State the value of the determinant of $\mathbf{C}$ and describe briefly how this value relates to the transformation represented by $\mathbf{C}$. **[2]**

*(Q5, Jan 2012)*

**46** The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

**(i)** Find the determinant of $\mathbf{X}$ in terms of $a$. **[3]**

**(ii)** Hence find the values of $a$ for which $\mathbf{X}$ is singular. **[3]**

**(iii)** Given that $\mathbf{X}$ is non-singular, find $\mathbf{X}^{-1}$ in terms of $a$. **[4]**

*(Q9, Jan 2012)*

**47** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find

**(i)** **AB**, **[2]**

**(ii)** $\mathbf{B}^{-1}\mathbf{A}^{-1}$. **[3]**

*(Q2, June 2012)*

**48** **(i)** The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **X**. **[2]**

**(ii)** The matrix **Z** is given by $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2+\sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1-2\sqrt{3}) \end{pmatrix}$. The transformation represented by **Z** is equivalent to the transformation represented by **X**, followed by another transformation represented by the matrix **Y**. Find **Y**. **[5]**

**(iii)** Describe fully the geometrical transformation represented by **Y**. **[2]**

*(Q9, June 2012)*

**49** The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

**(i)** Find the determinant of **D** in terms of $a$. **[3]**

**(ii)** Three simultaneous equations are shown below.

$$\begin{aligned} ax + 2y - z &= 0 \\ 2x + ay + z &= a \\ x + y + az &= a \end{aligned}$$

For each of the following values of $a$, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

**(a)** $a = 3$

**(b)** $a = 2$

**(c)** $a = 0$ **[7]**

*(Q10, June 2012)*

**50** The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and **I** denotes the $2 \times 2$ identity matrix. Find

**(i)** $2\mathbf{A} - 3\mathbf{I}$, **[3]**

**(ii)** $\mathbf{A}^{-1}$. **[2]**

*(Q1, Jan 2013)*

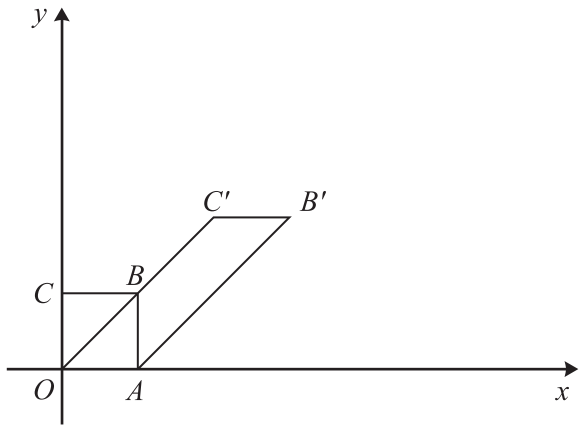
**51** By using the determinant of an appropriate matrix, find the values of $\lambda$ for which the simultaneous equations

$$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + \phantom{4}z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$

do not have a unique solution for $x$, $y$ and $z$. **[6]**

*(Q5, Jan 2013)*

**52**




The diagram shows the unit square $OABC$, and its image $OAB'C'$ after a transformation. The points have the following coordinates: $A(1, 0)$, $B(1, 1)$, $C(0, 1)$, $B'(3, 2)$ and $C'(2, 2)$.

**(i)** Write down the matrix, $\mathbf{X}$, for this transformation. **[2]**

**(ii)** The transformation represented by $\mathbf{X}$ is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. **[6]**

**(iii)** Find the matrix that represents transformation Q followed by transformation P. **[2]**

*(Q6, Jan 2013)*

**53** The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = (5 \quad 1)$, $\mathbf{B} = (2 \quad -5)$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

**(i)** Find $3\mathbf{A} - 4\mathbf{B}$. **[2]**

**(ii)** Find $\mathbf{CB}$. Determine whether $\mathbf{CB}$ is singular or non-singular, giving a reason for your answer. **[5]**

*(Q2, June 2013)*

**54** **(i)** Find the matrix that represents a rotation through 90° clockwise about the origin. **[2]**

**(ii)** Find the matrix that represents a reflection in the $x$-axis. **[2]**

**(iii)** Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the $x$-axis. **[2]**

**(iv)** Describe a **single** transformation that is represented by your answer to part **(iii)**. **[2]**

*(Q7, June 2013)*

**55** The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

**(i)** Find the value of $a$ for which $\mathbf{A}$ is singular. **[5]**

**(ii)** Given that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

**[7]**

*(Q10, June 2013)*